

# The Acquisition Argument - Intuitionism vs. Realism

Leopold Karl

October 2022

## Introduction

In this essay I will elaborate on the differences in the understanding of true and false between realists and intuitionists which is a result of their disagreement on semantics. As a specific example I will then consider the Acquisition Argument and argue that it fits intuitionistic semantics better than the realist ones.

## The understanding of true and false

As the realist position suggests that there are mathematical objects out there independent of us, they believe that every statement needs to be either true or false. This property of a mathematical sentence, namely having a truth value, is called truth-aptness in the philosophy of mathematics and is rejected by intuitionists since they believe that mathematics is created by us and hence, is dependent on us. As a result an intuitionist would only call a mathematical sentence truth-apt once he has decided upon its truth value: either it is true or it is false and, therefore, truth-apt or it is not even for certain that the statement has a truth value at all. For the intuitionist there simply is no gray-shaped area of truth-apt sentences without a specific truth value.

## The Acquisition Argument - the statement

The Acquisition Argument concerns how we become to speak the language of mathematics which is the most important tool for our learning, understanding and communication of mathematics. It states as follows: We are not born with a particular language of mathematics in mind, but rather need to learn it. We do so by being instructed or by using materials created by others who are already familiar with the language since there is no other way to obtain it. Furthermore, the resources we learn the language from need to follow some obtainable rules such that we end up pronouncing well-formed sentences instead of non-sense. Finally, we develop a determinate meaning of the language's sentences, however, we do so only knowing a limited, finite piece of what the whole language might be.

## The Acquisition Argument - in support of intuitionistic semantics

The first and main part of the Acquisition Argument which is how we learn the language of mathematics is incombable with the realistic point of view since if one believes that there are mathematical objects independent of us one cannot easily reject that there needs to be a certain way of obtaining them because assuming we cannot do so, talking about the existence of these objects would become meaningless. Hence, we shall all have the ability to obtain the existence of these mathematical objects birth-given, just as we are able to taste e.g. chocolate even though we might never have actually tasted it. However, this stands in contrast with the Acquisition Argument which states that the language of mathematics is only to be learnt from others. Of course, one might argue that obtaining mathematical objects needs practice as we seem to need practice whenever aiming to do something properly, but the realistic position implies the need of absolute rules for obtaining these objects and, as a result, the language of mathematics needs to be based on these

absolute rules. This not only contradicts the Acquisition Argument, but also our day-to-day notion of change in language, as well as the difference in use and perception of it. On the contrary, the intuistic position seems to be combinable with the Acquisition Argument's main statement. Since they believe in mathematics being created by us, it can easily be accepted to also use a language constructed and continuously developed by ourselves. Tackling the very last statement of the Acquisition Argument, the development of a determinate meaning of the language's sentences by each individual, it is even more straight forward that a realist is far more likely to reject to this idea, since this would suggest that each individual is able to obtain different truths or at least that there is an absolutely true perception and especially that all others aren't. On the opposite the intuistic position allows this individual understanding of mathematical language because if we construct mathematics by ourselves than there is no contradiction to be found in the mathematics someone else constructs out of his very own viewpoint and understanding of mathematics. Last, but not least, it just as well doesn't bother the intuitionist that a human is only able to obtain a finite number of mathematical objects and to know and understand a finite number of sentences since there really only is a finite number of these constructions. However, the realist should feel uncomfortable with the fact that he according to his position will never know mathematics as a whole.