

On structuralism

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1 Some words on structuralism

Even though the structuralist position can be split up into different streams, the idea all of them have in common is that mathematics is about investigating structures as opposed by the realist who claims to study real objects. Thereby, the structuralist avoids committing to an ontology and, hence, any critique concerning ontology, at least in the first place. Summing this up, the main idea behind structuralism is to study the relation between so-called concepts, which are objects introduced by a theory (like e.g. forces in physics) and, in particular, not believed to being real.

2 Structuralism's affinities to realism and formalism

A structuralist has a pretty similar approach concerning the day-to-day work of mathematicians as the realist does. They agree on what they study (mathematical objects) and which tools they use (e.g. intuition and concepts). Therefore, the structuralist and the realist position seem to be quite similar at first glance. However, as already mentioned in the introduction, the structuralist doesn't (necessarily) believe in the existence of mathematical objects and considers them being incidental. Consequently, the structuralist and the realist view differ on what lies behind mathematical objects. Nevertheless, finally the structuralist needs to decide on whether being realist or anti-realist on the existence of structures - another overlap between at least some streams of structuralism and realism.

Between structuralism and formalism there are several parallels to be found, aswell: First and foremost, most structuralist positions, such as Hellman's, do not forbid any theory. In particular, any structure is called a possible structure, which means its meaningful to study it from the structuralist's point of view, as long as it is consistent with the theory the structuralist works in, regardless of its consistency with other theories. This idea is highly linked to formalism, since they, aswell, suggest that there is meaning in studying any theory as long as it is consistent in itself. On the contrary, a realist would reject the idea of studying theories which would be inconsistent when merged, since they believe in a realm of mathematics. Another parallel between structuralism and formalism lies in their approach to mathematical objects: they both believe they are incidental or even avoid mentioning them. In particular, they both do not commit to an ontology.

In summary, the structuralist cleverly combines realism and formalism together with some rather new ideas. However, as we will notice in the next chapter, this combination also leads to a new problem structuralism is confronted with.

3 The structuralist's meta-structure problem

The arising problem for the structuralist is whether or not the structures he studies themselves are real and exist independently of us. Of course, one could start to argue that he was structuralistic (or even formalistic) about this question on the ontology of structures, but then the question of whether or not the structures of structures were real would arise and this infinite loop can only be escaped when committing to a realist or anti-realist position at some point. For taking the formalistic position it would be the same with the question of whether or not the chosen principles of conclusion are real (as discussed in former weeks). We will now consider three structuralist attempts of resolving this problem.

The first one is rather simple: just not dealing with this problem as e.g. Hellman does. However, a philosophical position which refuses to face a problem it itself arises cannot be regarded as a proper philosophical position.

The second approach is to take the anti-realist one. Even though not covered in the book I will now outline what I'd expect it to look like: Of course, there are various anti-realist positions, but what they all agree on is that there is no realm of mathematical objects existing independently of us. Therefore, the anti-realist position's line of argument would likely go into the direction of suggesting that structures do not exist independently of us and depending on the specific philosophical position one follows he would argue that structures are either something we invent and which afterwards exist (construcivism) or something we find in nature and which, if and only if we can find it there, exists (naturalism). (I left out formalism and structuralism here because they, as mentioned earlier, lead to the need of considering the next meta-level.)

The third approach is the one of a realist structuralist. He claims that structures do exist independently of us and, therefore, he should also be able to answer some follow-up questions such as e.g. how many structures there are. In short, the main issue of a realist structuralist is that he cannot allow the existence of a structure of all structures due to a similar argument as Russel's paradox. Hence, realist structuralism needs to be built on another mathematical theory, restricting it in such a way that the above mentioned paradox cannot occur. The resulting criticism on realist structuralism is mainly about its need of another mathematical theory as a basis.