The pluralists appeal to analytic proofs

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Introduction

In chapter 12 of her book „Pluralism in Mathematics“Michele Friend introduces the notion of axiomatic and analytic proofs and argues why every proof can and better be seen as analytic. This essay will focus on the definition of an analytic proof and the reasons for the pluralists suggestion to view proofs as analytic ones.

1 The analytic proof

An analytic proof is best described as a sequence of hypotheses which together prove the statement that is to be proven. The hypotheses themselves are proven aswell, but the tool box for proving these hypotheses is rather loosly defined: one could use inductive and deductive arguments as well as drawings, charts and even metaphors. To understand better what is so special about analytic proofs is probably to look at its counterpart: the axiomatic proof.

The axiomatic proof starts off with some axioms and derives the wanted statement by using a set of fixed rules (similar to what one would call a formal proof in the strict sense).

Even though one might argue that there the analytic proof leaves gaps, Friend argues in the mentioned chapter that there are gaps in the axiomatic proofs, too, and therefore, analytic proofs should not be seen as worth less than axiomatic proofs. To do so, she introduces the notion of internal and external gaps. Internal gaps appear in the analytic proof when a diagram or a metaphor is used to explain an argument since the argument then is not given in the mathematical language and (as e.g. the video of 3Blue1Brown suggests) this can even lead to „wrong“proofs or conclusions (meaning they actually haven't been proofs in the first place). On the contrary, external gaps have to do with how we choose our axioms or set of rules for deduction which is a similar problem to what a formalist faces when stating he wouldn't need to consider any ontology. However, it should be stressed at this point that no matter whether we talk of analytic or axiomatic proofs it is assumed by Friend that what we prove and the proof itself is correct, even though this seems a bit sketchy to me since the whole point of the discussion on what proofs are acceptable is about what norms to use to ensure this method only leads to correct proofs.

Why all proofs can be viewed as analytic

As next step towards her argumentation that proofs better be viewed as analytic, Friend first argues that all proofs can be viewed as analytic. The line of argumentation for this claim is rather simple. We distinguish between three kinds of proof:

Firstly, the informal and non-rigorous proof with no meta-proof. This one is analytic directly by the definition of an analytic proof.

Secondly, a proof for which there exist a meta-proof assuring that there also is an axiomatic proof (as it often appears in lectures due to time constraints). This meta-proof can either fall into the first or the third category, so we continue arguing as stated there.

Thirdly, if we have an axiomatic proof there again are two cases we want to distinguish between: On the one hand, maybe not all mathematicians agree with some of the axioms used in the proof and if so, there might arise internal gaps for them. Therefore, the proof can be seen as analytic. On the other hand, we might refuse to some mathematician's view and argue by stating that some formal systems are simply unreasonable (we kind of ban them by saying every mathematician woriking with/in them is not really a mathematician for us). However, our argumentation for banning these kinds of formal systems can only be reasoned by our „deep intuition"which is a poor rational argument since this intuition is exactly what is out for discussion.

Why proofs better be viewed as analytic

Finally, Friend argues why all proofs should better be viewed as analytic and gives three main reasons for this claim:
First of all, Friend argues that searching for an axiomatic proof does not help with actually finding a proof. In other words, if we chose the wrong path in our axiomatic proof at some point we are only going to notice that this is not the way to proof it, but don't receive any information about at which specific point of the proof we went wrong and therefore, don't know where to continue searching for a proof, while the analytic proof tells us where our proof failed and possibly also contains hided hints on what and where to change something to correct the proof.
Secondly, she states that finding axiomatic proofs is not what mathematicians do in practice. They rather try to understand more and more about the object they try to prove something about. To underpin this statement, some mathematicians are cited and all of them beat into the same notch: For his motivation the mathematician needs to intuitively know that what he wants to prove makes somehow sense and to be somehow familiar with the object of interest in the claim.
Thirdly, often, especially in teaching and in mathematical literature, the main point of a proof is to pass on an original idea and not to have the proof stored somewhere as a certificate for the statement being ultimately true. These ideas obviously cannot be passed on by giving axiomatic proofs since these would not communicate the most important points of the proof for catching the idea, would be overwhelming because of the quantity of information/material to read and to process and would take too long.

Conclusion

Pluralists see proofs as being analytic. In my opinion, this fits really good into the Pluralist approach to mathematics, since it once again is an attitude towards an important element of mathematics which prospers working together.