Essay: Non-Euclidean Arithmetic

based on Brian Rotman's text "Mathematics as Sign", Stanford: Stanford University Press, 2000

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1 Introduction

This week's reading "Mathematics as Sign" by Brian Rotman introduced me to a system of numbers which is very different from what we are used to work with, so in what follows I will, firstly, introduce this notion of numbers, called the Non-Euclidean Arithmetic, and, secondly, reason why Rotman proposes this new system.

2 The Non-Euclidean Arithmetic

The number system we work with (the naturals, rationals and reals as commonly constructed) has clearly been inspired by the rules we obtained from counting and their generalisations: associativity, commutativity, ordering, ...

However, when generalising our number system we do not need to assume that these rules hold for numbers which we will never reach by counting. In particular, we may not want to simplify our number system by assuming that there is someone öut there"who can count to any arbitrarily large number. This, an allowence for alternative number systems similar to the demand for Non-Euclidean Geometries, is where Rotman is going with his idea of Non-Euclidean Arithmetic.

Now, let me clarify what is meant by numbers which cannot be obtained by counting, the uncountable numbers: In theory, if we start counting from 1 we can (according to our current system) reach any (natural) number at some point. Notice, however, that this is not possible in reality, since we'd face time-constraints, run out of objects to count (e.g. more than atoms in the universe) or face energy-constraints, namely e.g. that it would cost a computer more energy to count up to a certain number (or even for the next +1 step) than there is in the visible universe. So since we constructed our current number system partly according to our real world experience (namely counting), we maybe would like to consider another, arguably even more natural system which also respects the mentioned real world constraints.

Let me now introduce such a system. For simplicity suppose that we can only count with one hand, meaning every (natural) number beyond 5 is uncountable. Now we want to have the ordinary calculation rules such as associativity, commutativity etc. for all countable numbers. However, we do not ask for uncountable numbers to fulfill these rules. As an example notice that $2 + 3 = 5 = 4 + 1 = 1 + 4$ still holds in this system. However, since 7 is uncountable we do not know $5 + 2 = 2 + 5$ nor $3 + 4 = 5 + 2$, since finding a one-to-one correspondence between $3 + 4$ and $5 + 2$ or $2 + 5$ is equivalent to counting to 7. Furthermore, this concept can also be extended to the rationals: Let a number $n = \frac{a}{b}$ be countable if both a, b are. Going back to our example we would notice that there is no countable number between $\frac{1}{4}$ and $\frac{1}{5}$. Hence, our number system would not only be restricted by an upper bound for countable numbers, but by an lower bound, aswell.

3 Rotman's reasons for proposing the Non-Euclidean Arithmetic

Even though the above example might seem and feel very strange, according to Rotman such a system applies to reality fairly well when taking the bound as big as one of the mentioned restrictions on counting given by the world surrounding us because any resource in the world is limited. Furthermore, Rotmann argues that such systems would "keep physics mathematically hones"(p. 135), since as already mentioned one cannot count arbitrarily far in the real world, and might help to understand the time in greater depth. Moreover, he hopes to provide a tool for investigating the opposition between serial and parallel actions with his Non-Euclidean Arithmetic.

4 Conclusion

This new number system seems to possibly become a great setting for further investigations in a lot of real world realted fields. However, it should be noted here that Rotman himself does not want to replace our current system with his, but rather consider problems in various systems to gain as much information as possible. Furthermore, he admits that this choice/allowence of alternative number systems and its possible applications are (yet) rather philosophical waters and yet to be explored in more depth.

These are 591 words.