Essay: What makes a mathematical statement

based on Roy Wagner's text "Conatus mathematico-philosophicus", in: Hetzel, Andreas, Schürmann, Eva and Schwaetzer Harald: Allgemeine Zeitschrift für Philosophie, 45(1), Stuttgart-Bad Cannstatt: Frommann-Holzboog, 2020, pp. 85–120.

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1 Introduction

This week's reading "Conatus mathematico-philosophicus" by Roy Wagner introduced me to Wittgenstein's philosophy of mathematics. In this essay I will answer the following questions: What differs a descriptive from a mathematical statement according to Wittgenstein? What makes a proposition mathematical in Wittgenstein's opinion?

2 Descriptive and mathematical statements

In his text Wagner distinguishes between descriptive statements and mathematical ones, also referred to as standards (I'll argue on that in "What makes a proposition mathematical", but for now we take these two expressions as synonyms.), according to how they are understood: A descriptive statement is one that accepted by the listener as it is, without questioning it's consistency, and that is (possibly) followed by an action of the listener that builds on the statement's results. In contrast, a standard or mathematical statement is a statement that is proved for consistency and correctness by the listener according to a certain pre-set norm. Notice that, therefore, a statement in itself is neither descriptive nor mathematical, but rather the listener makes it either of both according to how he reacts to the statement.

Let me now give a similar example to what has been discussed in the given text: The statement "there are 10 adults and 15 children coming to the party, so there we expect 30 guests", understood descriptively would mean that you as a chef would prepare 30 portions of lunch, since you consider the statement's result and act according to it. If you were to impose a mathematical standard, you would rather answer "wait a minute, 10 adults and 15 children don't make up to 30 guests", and, thereby, correct the statement according to the widely-spread norm of (standard) addition. However, I might insist "but I counted", to which you would reply "then you must have miscounted"änd even if I showed you a list of guests with 30 lines, all filled, you would try to search for the mistake (e.g. names appearing twice etc.), expecting me to be putting you on. Last, but not least, I might excuse myself by stating "well, it's 10 adults, 15 children and the five of us", to which you would reply by letting me know, that it's important to include that information into my statement to make it a reasonable one. You're committed to the mathematical standard.

3 Usage of mathematical standards

Although not a main topic of this essay, I would like to shortly mention some mathematical standards/some of their uses:

The names we give (natural) numbers are clearly a mathematical standard. Also, we can set a certain system as standard and always compare our results with this standard, as it is often done in physics: Physicists tend to compare their measured results to linear phenomena (give an error that occurs when compared to a linear model) and mostly describe space by how it deviates from the Euclidean model.

Furthermore, standards may mark limits of our conception as the following example shows: The standard ëvery finite collection can be counted"might seem intuitively true at first glance, but since it is unclear what should be able to count arbitrarily far in some absolute sense, a scenario (construction of a "counting-machine") without clever counter-example seems unlikely (as noted by Wagner in 1.2.4.1). Therefore, we use the mentioned standard because we cannot imagine an uncountable, finite collection, although logical reasoning cannot let us conclude its non-existence.

4 What makes a proposition mathematical?

While a statement can be viewed as either descriptive or as (setting) a standard, we tend to use the word mathematical for statements viewed the latter way. However, often they emerge from descriptions. This happens when we generalize observations we make, e.g. if you notice that after a lightning there comes a thunder and while at first you just state this as a description of a situation you experienced, you might set it as a standard after some occurrences. Still, you might experience a lightning that is not followed by a thunder somewhen, but since you do not consider the lightning-thunder relation as a description of some experiences you had any more, but as a standard, you will argue that you could not hear the thunder for some non-apparent reason, although there surely must have been a thunder following the lightning.

Another mathematical standard mentioned in Wagner's text is the Pythagorean Theorem. While emerged from descriptions, was proved and set as a standard in the mathematics of the ancient Greeks, it is nowadays only considered true in Euclidean spaces, so the scope at which we set the standard might change over time, meaning a standard might be contextualized.

Last, but not least let me remark, that a statement can really also be interpreted as both mathematical and descriptive, as the following example shows: "there are 10 adults, 15 children and the five of us at today's party, so please prepare 30 portions of lunch", can both be viewed as a description of a given situation and as setting the standard "$10 + 15 + 5 = 30$". How the speaker wanted the statement to be understood is not clear and might never be known. However, it may not matter. Altogether, as Wagner concludes, "mathematics comes about when some derivation techniques are used to set standards."(1.4.3)

These are 821 words.

Remark on the word count: I am sorry for writing too long. However, I publish most of my essays on my website and wanted to have the usage of mathematical standards included, so in case you don't find the time to read the whole essay/would deduct points for the essay being too lengthy, please ignore paragraph 3.